Instability of solitons in an inhomogeneous array of optical fibers

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The perturbation theory for an investigation of the stability of solitons in fiber array with a periodic change of a coupling constant is developed. The linear stability analysis is performed in an approximation of weak coupling within the framework of the system of dispersive discrete nonlinear Schrödinger equations. It is shown that the propagation of a soliton array in such a continuous-discrete system is unstable. The maximum of the growth rate of modulation instability is evaluated. Analyzing the mode structure of the corresponding eigenvalue problem in the vicinity of the threshold of instability it is found that in the system under consideration acoustical and optical unstable modes exist. Numerical calculations confirm the analytical results. [S1063-651X(98)13705-4]

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I. INTRODUCTION

Recently nonlinear fiber arrays and nonlinear waveguides have attracted a lot of attention in optics due to their possible applications in all-optical signal processing [1–5]. Moreover they provide an excellent opportunity for theoretical investigation of the nonlinear behavior of the discrete system. Another distinctive feature of these arrays is that they combine the properties of both discrete and continuous systems. For description of the pulse propagation in such systems the nonlinear discrete Schrödinger equation is usually used. Unfortunately this equation is not an integrable one, but it has soliton solutions realizing local extremum of the Hamiltonian. Such steady state solutions, corresponding to stationary points of the Hamiltonian, play an important role in system evolution. In general, in the search for the stationary localized solutions of nonlinear systems, inspection of their stability with respect to small perturbations is one of the fundamental problems of nonlinear physics. The detection of the soliton solutions, in turn, stimulates the development of powerful analytical and numerical methods through the examination of their stability. For the last several years, the evolution of localized and continuous wave solutions in the dispersionless nonlinear discrete systems has been studied intensively in many works (see, for example, [6–10], and references therein). But especially great interest is aroused by the investigations of the stability of the solitary waves in so-called continuous-discrete nonlinear systems where both the temporal dispersion and the discreteness are taken into account [11–15], because in this problem one may simultaneously investigate both the redistribution of the energy among the fibers and the evolution of the shape of the localized solution in each core. It was done in a pioneering work [15] where the modulation instability of soliton solutions in a homogeneous nonlinear fiber array was studied and, in particular, the threshold of this instability was evaluated. More recently, the stability of solitons, the phase of which rotates across homogeneous fiber array, was investigated in [16]. The influence of the inhomogeneity of the coupling strength on evolution of the solitary waves in the dispersionless fiber array was considered in [17–19].

In the present paper the stability of temporal solitons is studied in nonlinear fiber array including both dispersive properties of cores and inhomogeneity of the coupling strength. The change of the latter is caused by the periodic variation of linear coupling across the fiber array. The coupling coefficient can be modified, for example, by alteration of the separation between neighbor fibers. I examine in detail the array with an alternating variable part of the coupling coefficient from fiber to fiber. In practical applications the change of coupling coefficient can be done, for instance, by means of acoustical modulation of the distance between fibers.

The main objective of the present paper is a systematic linear stability analysis of the soliton array propagating in a linearly inhomogeneous continuous-discrete system. I demonstrate for the case of the set of two discrete dispersive nonlinear Schrödinger equations how weak coupling between neighbor channels changes the stability of the soliton array. The approach presented here was applied before mainly to continuum models (see, e.g., [20], and references therein). Unfortunately, as was specified by the authors of [20], the linear stability analysis of solitons in continuous media can be executed only in a limit of small spatial frequency. For the continuous-discrete model considered here the situation sharply varies. Below it will be shown that the analysis of stability of solitary wave solutions can be performed in an approximation of weak coupling with arbitrary spatial frequency of perturbations, that ensures the execution of the complete linear stability analysis of the homogeneous and inhomogeneous continuous-discrete systems within the framework of perturbation theory. Inspection of the eigenvalue problem in a perturbative manner allows one to obtain the spectrum of the soliton array perturbations. Analyzing the mode structure of the linearized system of differential equations of the fourth order I find that in the system considered two odd stable and two even slightly unstable modes exist. Modes corresponding to even branches of the spectrum will be termed optical and acoustical unstable modes. The discovered optical unstable mode is responsible for the process of modulation instability in the model of the soliton array under consideration.

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II. PERTURBATION THEORY

The evolution of pulses in an inhomogeneous array of coupled optical fibers in the anomalous dispersion regime is described by the system of differential equations

$$i\partial_z\Psi_n+C_{n+1/2}\Psi_{n+1}+C_{n-1/2}\Psi_{n-1}+\partial_{tt}\Psi_n+|\Psi_n|^2\Psi_n=0,$$
$$n=1,2,\ldots,N \tag{1}$$

where $C_{n+1/2}$ is the strength of coupling between fibers number n and $n+1$, Ψ_n is the complex amplitude in the nth core, t is the normalized retarded time, and z is the normalized propagation distance. In what follows I restrict myself to the consideration of the periodic change of coupling strength in the form

$$C_{n+1/2}=f+\Delta f\ \cos(\pi n), \tag{2}$$

where parameters f and Δf mean constant and variable part of this coupling coefficient, respectively. Now making a transformation of variables $\Psi_n\rightarrow\Psi_n e^{i(C_{n+1/2}+C_{n-1/2})z}$ and taking into account that the value of the coupling coefficient varies only with an interchange of parity of the number of fibers I rewrite Eq. (1) introducing the change of variables $\Psi_n=U_n$, for $n=2i$ and $\Psi_n=V_n$, for $n=2i-1$,

$$i\partial_z U_n+\Delta f(V_{n+1}-V_{n-1})+f(V_{n+1}+V_{n-1}-2U_n)+\partial_{tt}U_n$$
$$+|U_n|^2U_n=0, \tag{3}$$

$$i\partial_z V_n-\Delta f(U_{n+1}-U_{n-1})+f(U_{n+1}U_{n-1}-2V_n)+\partial_{tt}V_n$$
$$+|V_n|^2V_n=0. \tag{4}$$

This system of coupled equations is the Hamiltonian

$$H=\sum_n\int\Bigg\{f[|U_n-V_{n-1}|^2+|V_n-U_{n-1}|^2]$$
$$-\frac{\Delta f}{2}[\{U_n^*(V_{n+1}-V_{n-1})-V_n^*(U_{n+1}-U_{n-1})\}+\text{c.c.}]$$
$$+|\vec{\nabla}U_n|^2+|\vec{\nabla}V_n|^2-\tfrac{1}{2}|U_n|^4-\tfrac{1}{2}|V_n|^4\Bigg\}dt. \tag{5}$$

In terms of the Hamiltonian, Eqs. (3) and (4) are written as $i\partial U_n/\partial z=\delta H/\delta U_n^*$, $i\partial V_n/\partial z=\delta H/\delta V_n^*$. As usual, stationary equations may then be obtained from the variational approach

$$\delta(H+\lambda^2N_0)=0,$$

where $N_0=\Sigma_n\int\{|U_n|^2+|V_n|^2\}dt$ is the number of quanta and λ^2 plays the role of Lagrange multiplier. Consequently, soliton solutions

$$U_n^0=V_n^0=\frac{\sqrt{2}\lambda}{\cosh(\lambda t)}\,e^{i\lambda^2z}=g(t)e^{i\lambda^2z} \tag{6}$$

correspond to a stationary point of the Hamiltonian (5) with fixed number of quanta. Our goal is to consider the stability of the solitary waves (6) in the vicinity of this local extremum.

To study the effect of the weak linear coupling on stability of the soliton array, I develop the perturbation theory in the vicinity of the threshold of instability. In zero approximation, when one can neglect the interaction between neighbor channels, Eqs. (3) and (4) describe the propagation of the array of stable one-dimensional solitons (6). In this system of noninteracting solitons exists a set of neutrally stable modes corresponding to infinitesimal variations of soliton form. Moreover, if the linear interaction vanishes, the existence of these modes has no effect on the evolution of the system. Alternatively, an appearance of the small linear interaction between solitons leads to growth of certain previously neutrally stable perturbations and, as a consequence, to instability of the soliton system in array.

To analyze the stability of soliton solutions (6) with respect to the small perturbations I perform familiar linear stability analysis, where the equations for perturbations are derived by linearization of Eqs. (3) and (4) against the background of noninteracting solitons (6). Namely, the solution of the system (3) and (4) is looked for in the form $U_n=[g(t)+(\delta_1-i\delta_1')\cos(qn)+(\delta_3-i\delta_3')\sin(qn)]e^{i\lambda^2z}$, $V_n=[g(t)+(\delta_2-i\delta_2')\cos(qn)+(\delta_4-i\delta_4')\sin(qn)]e^{i\lambda^2z}$, where $\delta_i(t,z)$, $\delta_i'(t,z)$ are amplitudes of periodic perturbations with a spatial frequency q. For simplicity everywhere below it will be assumed that index i enumerates only the amplitudes and changes from 1 to 4. Then, substituting perturbed solutions U_n,V_n into Eqs. (3) and (4), keeping only linear terms over f, ΔfI reduce the initial system of evolution equations to four differential equations of the fourth order on t:

$$p^2\delta_1-\hat{L}_0\hat{L}_1\delta_1+2(\hat{L}_0+\hat{L}_1)(-f\delta_1+\Delta f\delta_4\sin q$$
$$+\delta_2 f\ \cos q)=0,$$
$$p^2\delta_2-\hat{L}_0\hat{L}_1\delta_2+2(\hat{L}_0+\hat{L}_1)(-f\delta_2-\Delta f\delta_3\sin q$$
$$+\delta_1 f\ \cos q)=0,$$
$$p^2\delta_3-\hat{L}_0\hat{L}_1\delta_3+2(\hat{L}_0+\hat{L}_1)(-f\delta_3-\Delta f\delta_2\sin q$$
$$+\delta_4 f\ \cos q)=0,$$
$$p^2\delta_4-\hat{L}_0\hat{L}_1\delta_4+2(\hat{L}_0+\hat{L}_1)(-f\delta_4+\Delta f\delta_1\sin q$$
$$+\delta_3 f\ \cos q)=0, \tag{7}$$

where z dependence of amplitudes of perturbations was assumed in the form $\delta_i\sim e^{ipz}$. Here $\hat{L}_0=\lambda^2-\Delta_{tt}-g^2$, $\hat{L}_1=\hat{L}_0-2g^2$ are the well-known self-conjugated operators of the Schrödinger type. Thus the problem of linear stability of the initial system of soliton array is reduced to the determination of the eigenvalue p of the spectral problem (7).

I suggest looking for the solution of the obtained system of differential equations of the fourth order (7) in perturbative manner, viz., expanding the solution in series of a weak coupling coefficient in vicinity of the threshold of instability. To do so, I take out among the linearly noninteracting perturbations with $f=0$ and $\Delta f=0$ the perturbations characterized by the eigenvalue $p=0$. These perturbations correspond to the neutrally stable modes describing infinitesimal variations of soliton parameters and can be considered as a zero approximation to the correct solution. This idea, forming the basis of perturbation theory, was put forward in [21] and successfully realized in the investigation of soliton stability in plasma and hydrodynamics [20]. But in continuum media, as mentioned above, it is impossible to extend the consequent linear stability analysis to arbitrary q. For the discrete system, as I am going to show below, the situation is essentially different. Really, when coupling constant f is small the eigenfunctions slightly differ from the neutrally stable modes for any q. This circumstance allows one to apply the perturbation theory for definition of the structure of the spectrum of the weakly coupling continuous-discrete system. Based on these arguments, the eigenfunctions δ_i as well as the eigenvalue p are expanded in the form of a series over a small parameter $f \ll \lambda^2$ as

$$\delta_i = \sum_{j=0}^{\infty} \delta_i^{(j)}, \quad p^2 = \sum_{j=1}^{\infty} (p^2)^{(j)}. \tag{8}$$

Then, the substitution of series (8) into Eq. (7) in zero order (i.e., for $p^{(0)}=0$, $f=0$, $\Delta f=0$) leads to

$$\hat{L}_0 \hat{L}_1 \delta_i^{(0)} = 0. \tag{9}$$

In this case the solution (6) describes the array of noninteracting solitons of the Schrödinger type. Generally speaking, each temporal solitary wave in array (6) is characterized by four parameters—initial phase, velocity, amplitude, and initial value of retarded time. As usual, the neutrally stable mode is defined by the difference between the stationary solutions with adjacent parameters. Hence, differentiating the stationary solution (6) with respect to the soliton parameters, one can directly obtain the following system of even and odd functions [20]:

$$\alpha^+ = -\frac{\partial g}{\partial \lambda^2}, \quad \alpha^- = \sqrt{2}\, \frac{\partial g}{\partial t}, \tag{10}$$

$$\beta^- = -\frac{tg}{\sqrt{2}}, \quad \beta^+ = g. \tag{11}$$

Alternatively, integrating Eq. (9) one may easily verify that solutions satisfying boundary condition $\delta_i^{(0)} \to 0$ as $t \to \pm\infty$ can be represented in the form of linear combinations of the neutrally stable modes (10)

$$\delta_i^{(0)} = C_i^+ \alpha^+ + C_i^- \alpha^-, \tag{12}$$

where C_i^+, C_i^- are arbitrary constants. As will be clear from further consideration, in the presence of weak interaction the odd and even parts of eigenfunctions δ_i correspond to $(p^2)^-$ and $(p^2)^+$ branches of the eigenvalue problem (7), respectively.

In this paper the stability analysis is restricted with accuracy up to the first order of perturbation theory. To receive the equations of the first approximation, it is necessary to substitute the series (8) into Eq. (7) and keep only the terms of the first order over $f, \Delta f$. It is evident that the solvability criterion of the obtained eigenvalue problem for the even branch of the spectrum "+" (or for odd "−") is expressed as

$$\langle \beta^{\pm} | \hat{L}_0 \hat{L}_1 | \delta_i^{(1)} \rangle = 0, \tag{13}$$

where brackets here and below denote the scalar multiplication. At first I will consider the even branch of the spectrum, corresponding to a sign "+" in Eq. (13). Substituting δ_i^1 from Eq. (7) into Eq. (13) and making use of Eq. (12) I arrive at

$$\begin{aligned}
C_1^+\left(p^2\,\frac{\langle\beta^+|\alpha^+\rangle}{2\langle\beta^+|\beta^+\rangle} - f\right) + C_4^+ \Delta f \sin q + C_2^+ f \cos q &= 0,\\
C_2^+\left(p^2\,\frac{\langle\beta^+|\alpha^+\rangle}{2\langle\beta^+|\beta^+\rangle} - f\right) - C_3^+ \Delta f \sin q + C_1^+ f \cos q &= 0,\\
C_3^+\left(p^2\,\frac{\langle\beta^+|\alpha^+\rangle}{2\langle\beta^+|\beta^+\rangle} - f\right) - C_2^+ \Delta f \sin q + C_4^+ f \cos q &= 0,\\
C_4^+\left(p^2\,\frac{\langle\beta^+|\alpha^+\rangle}{2\langle\beta^+|\beta^+\rangle} - f\right) + C_1^+ \Delta f \sin q + C_3^+ f \cos q &= 0,
\end{aligned} \tag{14}$$

where the value $\langle\beta^+|\beta^+\rangle / \langle\beta^+|\alpha^+\rangle = -4\lambda^2$ can easily be calculated. Thus the approach applied here allows conversion of a complicated system of differential equations of the fourth order to a trivial system of algebraic identities. After the simple evaluations I get the dispersive relation between the wave number p^+ and the perturbed spatial frequency q,

$$(p^2)_{1,2}^+ = 8\lambda^2 f \left\{ -1 \pm \left[1 - \left(1 - \frac{(\Delta f\,)^2}{f^2} \right) \sin^2 q \right]^{1/2} \right\}. \tag{15}$$

Here indices 1,2 correspond to signs "+" and "−" at the radical, respectively. Equation (15) shows that there are two unstable even branches of the spectrum and that the soliton array is unstable for any q. The corresponding structure of normalized growth rate $\gamma = p^+/\lambda\sqrt{8f}$ of the aperiodic instability (15) in the first Brillouin zone is displayed in Fig. 1. First, note that for the upper branch of instability in a point $q=0$ the p^2 reaches its maximum negative value ($\gamma_{\max}^2 \approx 16 f \lambda^2$), whereas for the lower-lying branch of instability

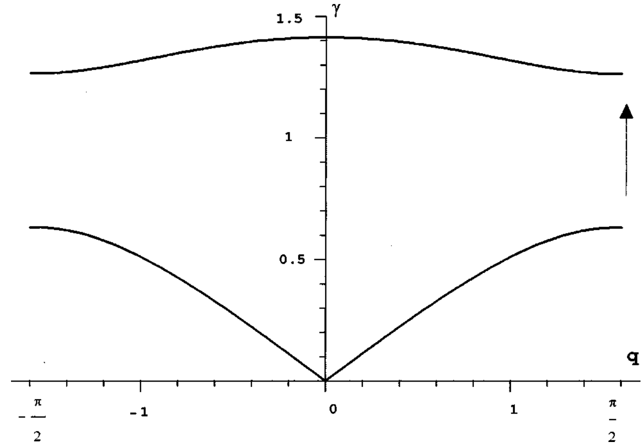

FIG. 1. Normalized growth rate of instability $\gamma = p^+/\lambda\sqrt{8f}$ vs the spatial frequency of perturbations q in the first Brillouin zone, where $\Delta f/f = 0.3$. An arrow on the boundary of the Brillouin zone at $q=\pi/2$ indicates a gap in the spectrum equal to $\sqrt{2\Delta f/f}$.

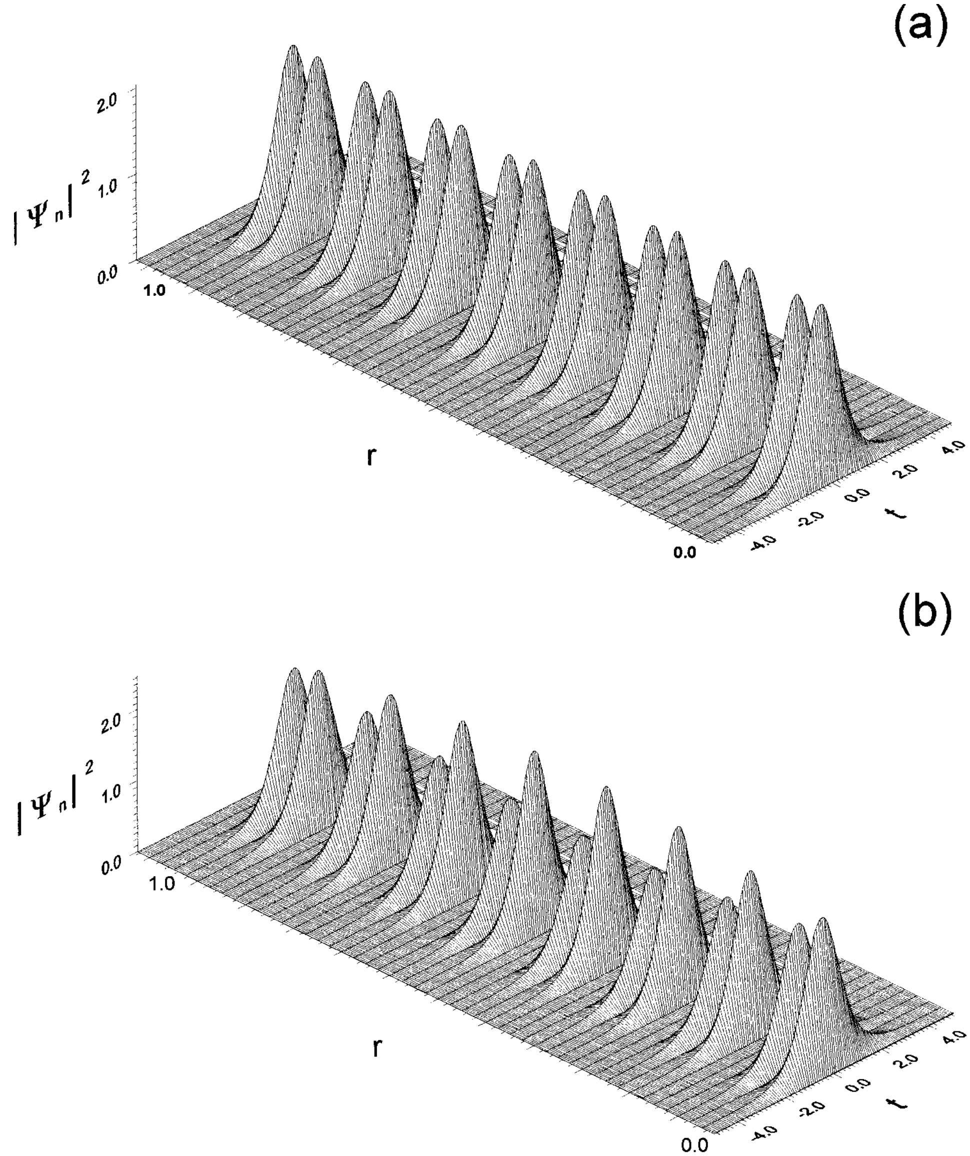

FIG. 2. Distribution of energy $|\Psi_n|^2$ in array after propagation distance z with parameters of coupling coefficient $f=0.1$, $\Delta f=0.01$. (a) $z=0$. (b) $z=4$. (c) $z=6$. (d) $z=8$.

in this point the p attains to zero. Then the minimum distance between various branches of the spectrum is achieved on the edges of the first Brillouin zone for $q=\pm\pi/2$ and is equal to $4\lambda\sqrt{\Delta f}$. If Δf tends to zero the gap in a spectrum disappears and Eq. (15) is reduced to the case of the homogeneous soliton array $(p^2)^+=-16\lambda^2 f\sin^2(q/2)$ [16]. Moreover, in the limit $q\rightarrow 0$ (i.e., in the semidiscrete approximation) the well-known result $p^2=-4\lambda^2 fq^2$ may then be obtained [20]. Analysis of Eq. (15) shows that the model considered has a new branch of a spectrum (upper branch in Fig. 1) which cannot be obtained in the homogeneous limit and which will be termed further as an optical unstable branch. The even branch of the spectrum corresponding to a sign "+" in Eq. (15), when p is proportionally q for small q, may then be termed, apparently as an acoustical unstable branch. For small q it is not difficult to define the ratio of the amplitudes of perturbations for the optical branch of the spectrum, namely, $\delta_1/\delta_2=\delta_3/\delta_4=-1$. Obviously, the appearance of this optical unstable branch of the spectrum is caused by the inhomogeneity of the continuous-discrete system considered. Thus the periodic variation of the coupling coefficient gives rise to an eigenvalue structure with band gap.

Repeating the above described procedure for the odd branch of a spectrum I get

$$(p_{1,2}^-)^2=\frac{8}{3}\lambda^2 f\left\{1\pm\left[1-\left(1-\frac{(\Delta f)^2}{f^2}\right)\sin^2 q\right]^{1/2}\right\}. \tag{16}$$

In Eq. (16) a value of $\langle\alpha^-|\alpha^-\rangle/\langle\beta^-|\alpha^-\rangle=\frac{4}{3}\lambda^2$ was evaluated using the form of the soliton solution. Analyzing the relation (16) I find that the structure of these two odd branches of the spectrum is similar to the structure of the acoustical and optical branches of a linear lattice with different masses. At first sight, it may seem paradoxical or at least surprising, but it is necessary to notice that the symmetry of the coupling coefficient is similar to the symmetry of the

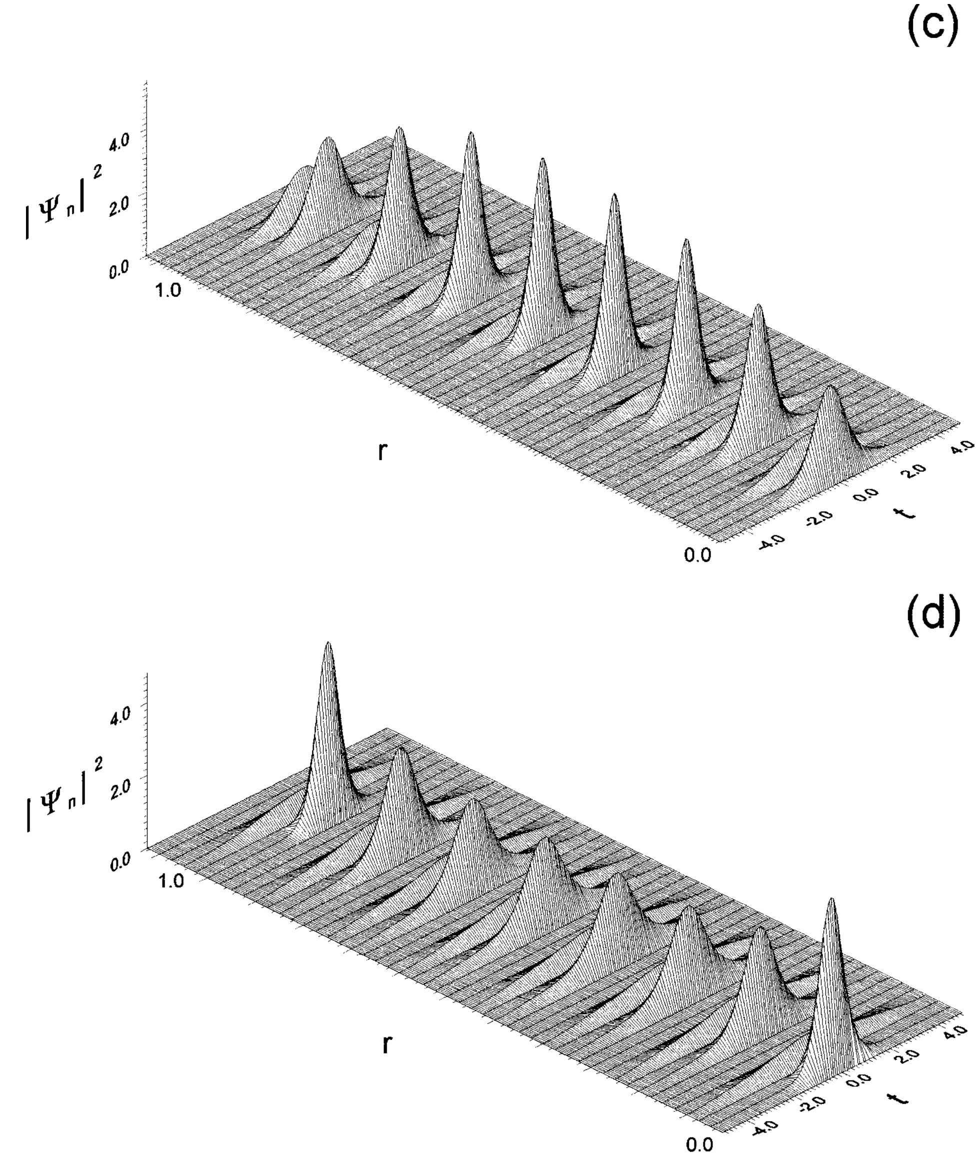

FIG. 2. (*Continued.*)

linear chain, consisting of particles of two sorts with the different masses [see Fig. 2(a)]. Continuing this analogy further one can easily identify the magnitudes f and Δf as a sum and difference of inverse masses, respectively. An especially interesting correspondence may be obtained if it is accepted that the coupling constant is inversely proportional to a distance between adjacent fibers. In this case the distance between neighboring fibers can therefore be associated with the mass of a particle in the linear lattice and the above-mentioned correspondence with the linear chain becomes especially clear. The magnitude $2\langle\alpha^-|\alpha^-\rangle/\langle\beta^-|\alpha^-\rangle$ reflects the nature of the objects considered (i.e., solitons) and plays in such an analogy the role of a spring constant.

Now, a remark is in order. In the above-described stability analysis the perturbed function constitutes an eight-parameter family of amplitudes of periodic perturbations. As a matter of fact, in the system under consideration four special four-parameter substitutions exist. This set of periodic perturbations lets me represent for compactness in the form of two vectors

$$\delta U_n\begin{pmatrix}1\\1\\1\\1\end{pmatrix}=\left[\begin{pmatrix}1\\1\\-1\\1\end{pmatrix}(\delta_1-i\delta_1')\cos(qn)+\begin{pmatrix}1\\1\\1\\-1\end{pmatrix}\times(\delta_2-i\delta_2')\sin(qn)\right]e^{i\lambda^2 z},$$

$$\delta V_n\begin{pmatrix}1\\1\\1\\1\end{pmatrix}=\left[\begin{pmatrix}-1\\1\\1\\1\end{pmatrix}(\delta_1-i\delta_1')\cos(qn)+\begin{pmatrix}1\\-1\\1\\1\end{pmatrix}\times(\delta_2-i\delta_2')\sin(qn)\right]e^{i\lambda^2 z},$$

with the components $\delta U_n = U_n - g(t)e^{i\lambda^2 z}$ and $\delta V_n = V_n - g(t)e^{i\lambda^2 z}$. One easily verifies that these perturbed functions are also solutions of the linearized system of evolution equations and moreover realize spectra (15) and (16).

As was already mentioned above, the represented analysis of stability is valid, of course, providing that $f \ll \lambda^2$. Nevertheless, the stability of the initial soliton solution (6) may be rather crudely estimated for the case $f > \lambda^2$ as well:

$$p^2 = 8f^2 - 4[f^2 - (\Delta f\,)^2]\sin^2 q \pm 8f^2\left[1 - \left(1 - \frac{(\Delta f\,)^2}{f^2}\right)\sin^2 q\right]^{1/2}. \tag{17}$$

As follows from Eq. (17) the value p^2 is nonnegative and therefore the range of instability of soliton oscillations is limited by $f \sim \lambda^2$. Hence, for arbitrary f the maximum of the growth rate is given by $\gamma_{\max} \approx \lambda^2$, which is of the same order as the growth rate of the modulation instability of the monochromatic wave.

III. NUMERICAL RESULTS

The stability of the weak coupling continuous-discrete system (3) and (4) was checked numerically as well. In the preceding section Eq. (15) showed that the soliton solution (6) is unstable with respect to small periodic perturbations. In numerical simulations for a nonlinear array of 16 fibers the split-step method was used, where the linear coupling term as well as the nonlinear term were calculated exactly, while the linear dispersive term was evaluated with a fast-Fourier-transform algorithm with up to 128 temporal grid points. In numerical calculations both the absorbent and periodic boundary conditions were examined. In a weak coupling approximation the evolution of the soliton system weakly depends on the type of boundary conditions. So, in Figs. 2(a)–2(d) the numerical results only for periodic boundary conditions are presented. Previously presented theoretical analysis shows that the perturbations with a small spatial frequency, corresponding to the optical unstable mode, are most unstable, so that for numerical simulations the initial conditions [see Fig. 2(a)] were taken in the form $\Psi_n(0,t) = \sqrt{2}\mu_n/\cosh(\mu_n t)$, where $\mu_n = 1 - (-1)^n \epsilon \sin(\pi n/16)$ with the amplitude of perturbations $\epsilon = 0.01$. On an axis r is shown the location r_n of the nth fiber in the array determined by relations $r_1 = r_{\text{initial}}$, $r_n = r_{n-1} + r_0 + (-1)^n \Delta r$ for $n = 2,...,16$, where every r_n is scaled to $r_{16} = 1$ and the distances between neighbor fibers $r_0 - \Delta r$ and $r_0 + \Delta r$ correspond to the interactions $f + \Delta f$ and $f - \Delta f$, respectively. The oscillating Δf and constant f parts of the coupling parameter have been varied in wide limits within from 0.001 to 0.1. In Figs. 2(a)–2(d) are displayed the results for $\Delta f = 0.01$ and $\Delta f = 0.1$, as an example.

Numerical calculations show that in the process of soliton evolution a few stages may be chosen. The first stage is characterized by growth of initial modulations [see Fig. 2(b)]. As could be anticipated, the even unstable optical mode provokes a bunching of initial soliton array in the linearly inhomogeneous continuous-discrete system under consideration [15,20]. Because of the smallness of the discrete dispersion in the process of creation of one new localized entity, only a small part of the energy of the soliton array is involved. Really, in this scenario the period of the structure obtained (i.e., the distance between localized states containing a considerable amount of energy) is increased only twice, in contrast with the initial state. At the end of this stage the energy almost completely concentrates in solitons with large amplitudes. On the next stage are observed the oscillations of the energy between localized states obtained with big spikes. Some snapshots of this stage are sketched in Figs. 2(c) and 2(d). Numerical simulations show also that for small perturbed frequency the growth rate of instability practically does not depend on Δf in accordance with the analytical results (15). It should be noted that the magnitude of the coupling constant f plays the pivotal role in the evolution of the inhomogeneous continuous-discrete system considered. My computer simulations show also that with decrease of the distance between fibers the number of spikes, containing most of the energy, is decreased as well, so that for $f \sim 1$ almost all of the energy of the array is contained in one or a few fibers, but that is beyond the goals of this paper.

IV. CONCLUSIONS

In conclusion, linear stability analysis of the array of solitons in the linearly inhomogeneous continuous-discrete nonlinear system was performed by means of the perturbation theory presented here. The approach applied here allowed me to obtain in a perturbative manner the solution of the system of linearized evolution equations containing differential operators of the fourth order. The smallness of interaction between solitons has provided, in the vicinity of the threshold of instability, the possibility to seek eigenvalues and eigenfunctions of the corresponding spectral problem as an expansion over a small interaction coefficient. In the model under consideration inhomogeneity was initiated by periodic change of the coupling strength. The effect of the coupling strength variation may be seen as the opening of a gap in the eigenvalue spectrum. Analysis of eigenvalue structure near the threshold of instability shows that the spectrum of perturbations of the soliton array in linear approximation over coupling coefficient is similar to the spectrum of acoustical and optical oscillations of a linear discrete chain. But in an inhomogeneous fiber array, in addition to odd stable modes, slightly unstable even modes exist, which were referred to above as optical and acoustical slightly unstable modes in analogy with oscillations of a one-dimensional lattice. The discovered optical unstable mode ensures instability of the soliton array in the model considered and vanishes completely in the homogeneous limit when $\Delta f = 0$. Numerical calculations have demonstrated the development of modulation instability of an inhomogeneous fiber array for different boundary conditions. The analytical method presented can be extended for linear stability analysis of other configurations of soliton arrays in different models of fiber arrays that will be presented elsewhere.

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